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"ANALYSIS ON MATHEMATICAL MODEL FOR TWO PHASE (NON- NEWTONIAN & NEWTONIAN) BLOOD FLOW IN CAPILLARY WITH SPECIAL REFERENCE TO ASTHMA FOR A SPECIAL CASE, ALLAHABAD (U.P), INDIA"

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ABSTRACT

The main aim of present study is to examine a Non-Newtonian mathematical model of two phased blood flow in human pulmonary capillary, keeping in view the nature of pulmonary blood circulation. Herschel Bulkley Non – Newtonian model in Bio – fluid mechanical setup is applied with respect to the help of clinical data in case of Asthma for hemoglobin versus blood pressure. In present study overall presentation is in tensorial form and the solution technique adopted is analytical as well as numerical.

KEYWORDS: *Pulmonary Blood Flow, Herschel Bulkley, Non – Newtonian Model, Plasma Layer, Hematocrit, Power-Law Model, Blood Pressure Drop, Behaviour of Blood*

Article History

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INTRODUCTION

1. Structure and Function of the Lungs

The human lungs are paired organs in the chest and divided in two lobes. Where the right lung has three lobes, the left one has two lobes. It is smaller to accommodate to the heart, which takes up the space of the third lobe in the chest cavity.

Figure 1

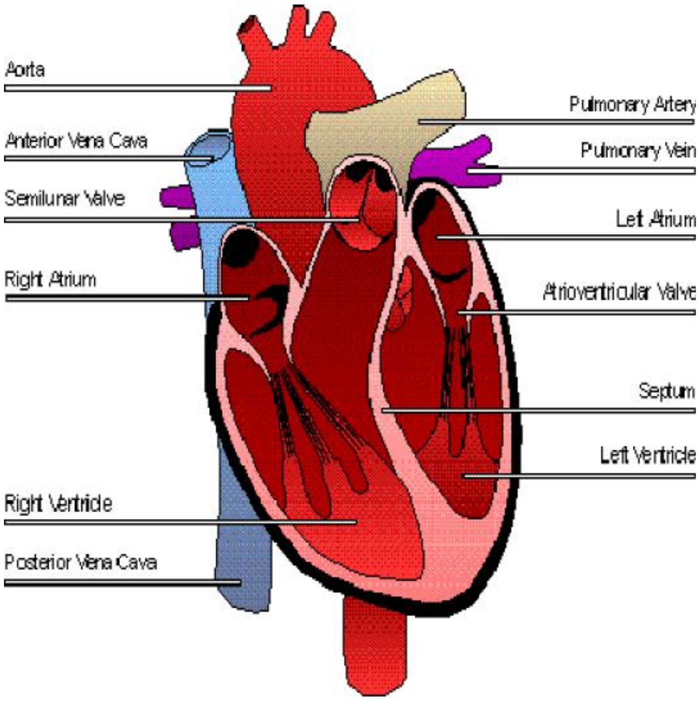

Figure 2.

Figure 3.

2. Mathematical Modeling (Choice of Frame of Reference)

A frame of reference on mathematical modeling for moving blood we observed by the three - dimensional orthogonal curvilinear co-ordinate system, briefly prescribed as E3,called as 3-dim Euclidean space (tensorial form).

2.1. Equation of Continuity

Let the volume portion covered by blood cells in unit volume be X, this X is replaced by H/100, where H is the Hematocrit the volume percentage of blood cells. Then the volume portion covered by the plasma will be 1-X. If the mass ratio of blood cells to plasma is r then clearly.

$$r = \frac{Xpc}{(1-X)Pp} \tag{1}$$

where, Pc and Pp are densities of blood cells and blood plasma respectively.

The both phase of blood, move with the common velocity. Hence, the equation of continuity for two phase by the principle of conservation of mass is

$$\frac{\partial(Xpc)}{\partial t} + (XPcV^i), i = 0 \tag{2}$$

And

$$\frac{\partial(1-X)Pp}{\partial t} + (1-X)Ppv^i), i = 0 \tag{3}$$

Where, v is the common velocity of two phase blood cells and plasma. Again ($Xpcv^i$) is co-variant derivative of ($Xpcv^i$) with respect to X^i. In the same way (1-X)*P*c V^i), i is co-variant derivative of ((1-X)*p*cV^i) w.r. to X^i

If we define the uniform density of the blood *Pm* then we consider

$$\frac{1+r}{p_m} = \frac{r}{p_c} + \frac{1}{Pp} \tag{4}$$

From equations (2) &(3) we consider

$$\frac{\partial p_m}{\partial t} + \left(Pmv^i\right)i = 0 \tag{5}$$

2.2. Equation of Motion

The equation of motion for the two phase of blood cells is

$$Xpc\frac{\partial vi}{\partial t} + \left(Xp_cV^j\right)V^i_j = -Xp, jg^{ij} + X\eta c\,(g^{jk}\,V^i_k), j \tag{6}$$

Similarly, the equation of motion for plasma is

$$(1-X)Pp\frac{\partial v^i}{\partial t} + \{(1-X)PpV^i\}V^i_j = -(1-X)p, jg^{ij} + (1-X)p, jg^{ij} + (1-X)\eta c\,(g^{jk}V^i_k), j \tag{7}$$

Now, adding equations (6) & (7) and by relation (4), the equation of motion for blood flow with the both phases will be

$$Pm\frac{\partial v^i}{\partial t} + \left(PmV^j\right)v^i_j = -p,\ j + \eta_m(g^{jk}\,v^i_k), j \tag{8}$$

Where $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity coefficient of blood as a mixture of two phase. As the velocity of blood flow decreases, the viscosity of blood increases. In this situation, the blood cells line up on the axis to build up rouleaux. Hence a yield stress is produced. Though this yield stress is very small, even then the viscosity of blood is increased nearly ten times. The Herschel Bulkley law holds good on the two phase blood flow through veins Lungs, veinules and whose constitutive equation is

$T' = \eta_m e^n + T_p \ (T' \geq T_p)$ and

$e = o\,(T' < T_p)$ where, T_p is the yield stress. When strain rate e = o $(T' \geq T_p)$ a core region is formed which flows just like a plug. Let the radius of the plug be r_p. The stress acting on the surface of plug will be T_p. Equating the forces acting on the plug, we get,

$$P\pi_p^{\ 2} = T_p\, 2\,\pi r_p$$

$$\Rightarrow rp = 2\frac{Tp}{P} \tag{9}$$

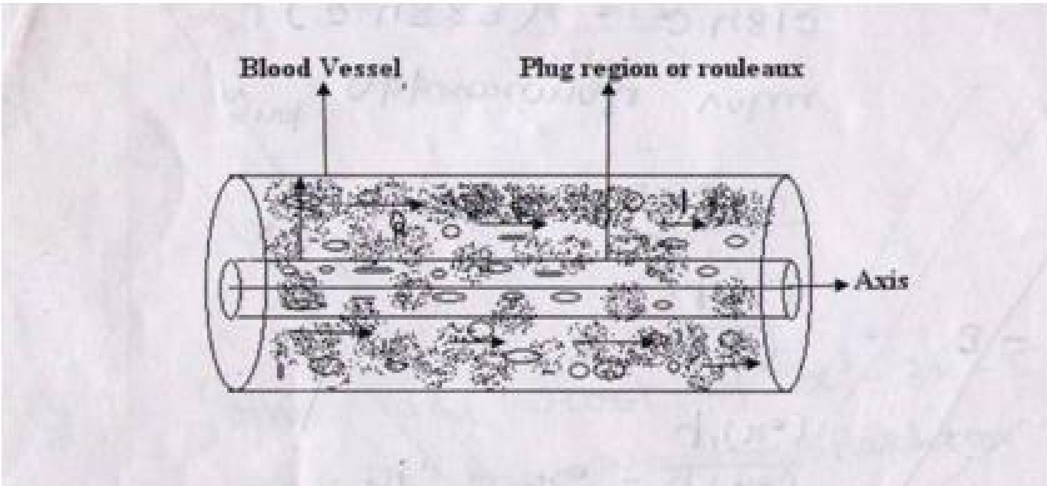

Fig (1): Herschel Bulkley blood flow

Figure 4

The Constitutive equation for test part of the blood vessel is

$$T' = \eta m e^n + T_p \quad \text{Or } T' - T_p = \eta_m e^n = T_e$$

Where, Te = effective Stress

Whose generalized form will be

$$T^{ij} = -P g^{ij} + T_e^{ij} \quad \text{where } , T_e^{ij} = \eta_m (e^{ij})^n \quad \text{While } e^{ij} = g^{jk} V_k^i$$

Now we describe the basic equation for Herschel Bulkley blood flow then

(i). Equation of Continuity

1 /   g (gvi),= 0.

(ii) Equation of Motion

$$p_m \frac{\partial vi}{\partial t} + PmV^iV^i{}_{,j} = -T_{e,j}{}^{ij} \tag{10}$$

Where, all the symbols have their usual meanings.

2.3. Analysis

Since, the blood vessels are cylindrical; the above governing equations have to be transformed into cylindrical co-ordinates.

$$X^1 = r,\ X^2 = \overline{\ },X^3 = Z,$$

Matrix of metric tensor in cylindrical co-ordinates is

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

While matrix of conjugate metric tensor is

$$[g^{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where, as the Chritoffel's symbols of 2nd kind are

$$\begin{Bmatrix} 1 \\ 2 \quad 2 \end{Bmatrix} = -r, \begin{Bmatrix} 1 \\ 2 \quad 2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \quad 2 \end{Bmatrix} = \frac{1}{r}$$

Remaining others are zero.

Relation between contra variant & physical components of velocity of blood flow will be

$$\sqrt{g_{11v}{}^1} = v_r \Rightarrow v_r = v^1$$

$$\sqrt{g_{22\,v}{}^2} = v_{-} \Rightarrow v_{-} = r\, v^2$$

$$\sqrt{g_{33\,v}{}^3} = v_z \Rightarrow v_z = v^3$$

Again the physical components of $-p_{,j}g^{ij}$ are - $\sqrt{g_{ij}}\ P_{,j}\ g^{ij}$

Equation (9) and (10) are transformed into cylindrical form so as to solve them as power law model then

$$\frac{dv}{dr} = \left(\frac{\Pr}{2\eta_m}\right)^{\frac{1}{n}}$$

Where, pressure gradient $\frac{dp}{dz} = P$

$$\frac{dv}{dr} = \left(\frac{p(r - r_p)}{2\eta_m}\right)^{\frac{1}{n}}$$

$$\frac{dv}{dr} = \left(\frac{\frac{1}{2}pr - \frac{1}{2}pr_p}{\eta_m}\right)^{\frac{1}{n}}$$

by equation (9)

$$\frac{dv}{dr} = \left(\frac{\frac{1}{2}pr - T_p}{\eta_m}\right)^{\frac{1}{n}} \tag{11}$$

Substituting the value of Tp from (7) into (11), then

$$\frac{dv}{dr} = \left(\frac{\frac{1}{2}pr - \frac{1}{2}\Pr_p}{\eta m}\right)^{\frac{1}{n}}$$

$$\frac{dv}{dr} = -(p/2\eta m)^{\frac{1}{n}}\ (r - r_p)^{\frac{1}{n}} \tag{12}$$

Integrating under the no slip boundary condition: v = 0 at r = R then we have

$$V = (P/2\eta_m)^{\frac{1}{n}}\ \frac{n}{n+1}\left[(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1}\right] \tag{13}$$

This is the formula for velocity of blood flow in Lungs veinules and veins.

Putting $r = r_p$ then the velocity V_p of plug flow is

$$\mathrm{V_p} = \frac{\mathrm{n}}{\mathrm{n}+1}\ (\mathrm{P}/2\eta\,\mathrm{m})^{\frac{1}{\mathrm{n}}}(\mathrm{R} - \mathrm{r_p})^{\frac{1}{\mathrm{n}}+1} \tag{14}$$

2.4. Result

Observations

Hematocrit vs. blood pressure from an authorized. Homeo Chikitsa Kendra, Allahabad by Dr. K. K. Mishra.

Patient: Pyare lal

Diagnosis: Asthma.

B-H Table

Table 1

S. N	Date	HB(Hemoglobin)	B.P. (Blood Pressure)	Hematocrit	Hemoglobin
1	10/9/20	12.2	140/100	36	12
2	16/9/20	11.6	130/90	35.4	11.8
3	20/8/20	11.4	130.3/80	33.6	11.2

The flow flux of two phased blood flow in Lungs, veinules and veins is

$$Q = \int_0^{r_p} 2\pi r v_p \, dr + \int_{r_p}^{R} 2\pi r v \, dr$$

$$= \int_0^{r_p} 2\pi \frac{n}{n+1}\left(\frac{p}{2\eta m}\right)^{\frac{1}{n}} (R-r_p)^{\frac{1}{n}+1} \, dr +$$

$$\int_0^{r_p} 2\pi \frac{n}{n+1}(P/2\eta m)^{\frac{1}{n}}\left[(R-r_p)^{\frac{1}{n}+1} - (r-r_p)^{\frac{1}{n}+1}\right] dr$$

Solving (12) & (14) then

$$\frac{2\pi n}{(n+1)}\left(\frac{P}{2\eta m}\right)^{\frac{1}{n}}(R-r_p)^{\frac{1}{n}+1}\left[\frac{r^2}{2}\right]_0^{r_p} + \frac{2\pi n}{(n+1)}\left(\frac{P}{2\eta m}\right)^{\frac{1}{n}}\left[\frac{r^2}{2}(R-r_p)^{\frac{1}{n}+1} - \frac{r(r-r_p)^{\frac{1}{n}}}{\frac{1}{n}+2} + \frac{(r-r_p)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)}\right]_{r_p}^{R}$$

Q = 425.136 ml. / min

R = 1, r_p =.999

¦ $_p$ = 0.0013 (Pascal - sec.)

¦ $_m$ = 0.027 (Pascal- sec.)

.H = 24

$$\eta_m = \eta_c X + \eta_p (1-X) \text{ where, } X = \frac{H}{100} = \frac{24}{100} = .24$$

$$1827.23 = (1728.40)^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

Solved by Numerical method, then **n = 1.127**

$$\Delta P = (.003213H + .0039)(1827.23)^{1.15} \times \left(\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right)^n$$

H	36.1	35.4	33.6	33.2
ΔP	193.312	190.193	180.843	177.721

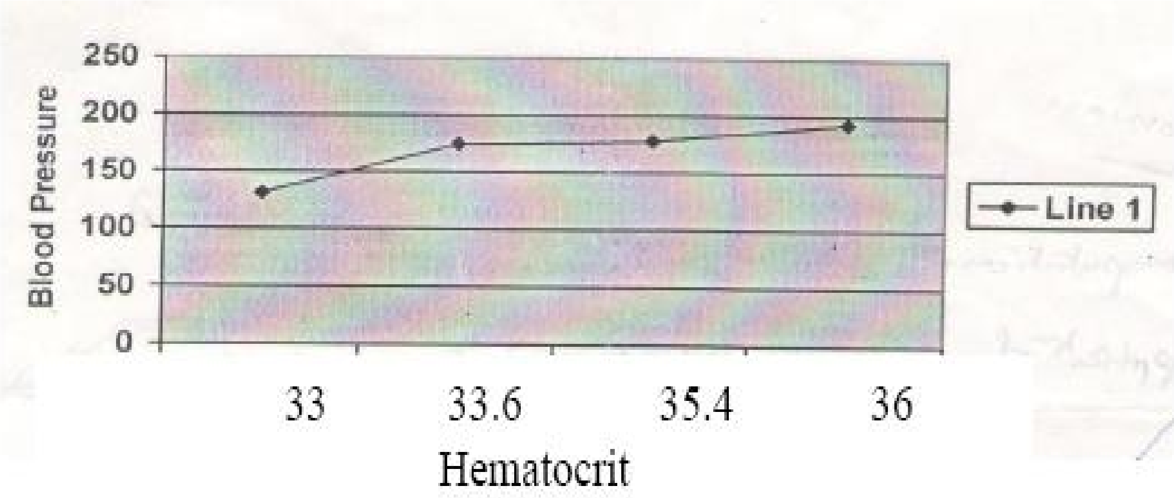

Figure 5

CONCLUSION

It is clear that when Hematocrit is increased the Blood pressure also increased. Hence Hematocrit is proportional to blood pressure.

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